Code No: 131AB **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, September - 2023

MATHEMATICS - II

(Common to CE, ME, MCT, MMT, AE, MIE, PTM, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Find the Laplace transform of $e^{at}\sin bt$. [2]

b) Find the Laplace transform of $v(t)=\begin{cases} t, & 0<t<1 \\ 1, & t>1 \end{cases}$. [3]

c) Find the value of $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$? [2]

d) Find the value of $\int_0^1 x^3(1-\sqrt{x})^5$ using Beta and Gamma functions. [3]

e) Find the area bounded by the curves $y=0, y=1,\ x=1$ and $x+y=2$? [2]

f) Find the double integral of $f(x,y)=\frac{x}{y}$ over the region in the first quadrant bounded by the lines $y=x,\ y=3x, x=2$ and $x=3$? [3]

g) Evaluate $div\ \overline{F}$ at the point (1,2,3) given $\overline{F}=x^2yz\overline{i}+xy^2z\overline{j}+xyz^2\overline{k}$. [2]

h) Evaluate curl $\overline{F}$ at the point (1,2,3) given $\overline{F}=3x^2\overline{i}+5xy^2\overline{j}+5xyz^3\overline{k}$. [3]

i) Show that the vector $(x^2-yz)I+(y^2-zx)J+(z^2-xy)K$ is irrotational. [2]

j) Find the area bounded by the curves, $y=1,\ x=1, x=e^y$ and $x=e$. [3]

PART - B

(50 Marks)

2.a) Prove that $L^{-1}\left\{\frac{2a^2s}{s^4-a^4}\right\}$ by using the convolution theorem.

b) Solve an ordinary differential equation $y''+y'=\cos t$ when $y(0)=0=y'(0)=0$. [5+5]

OR

3.a) Find the Laplace transform of f(t) =t/T for $0\le t\le T$ and $f(t)$ is periodic with period T.

b) Find $L^{-1}\left\{\frac{a^3}{s^2\left(s^2+a^2\right)}\right\}, s>0.$ [5+5]

4.a) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

b) Evaluate $\int_0^{\pi/2}\sqrt{tanx}dx$. [5+5]

OR

5. Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^4)}} \times \int_0^1 \frac{dx}{\sqrt{(1+x^4)}} = \frac{\pi}{4\sqrt{2}}$. [10]

6.a) Evaluate $\iint \frac{r dr d\theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$.

b) Using double integration, find the Centre of Gravity of a lamina in the shape of a quadrant of the curve $(x/a)^{2/3} + (y/b)^{2/3} = 1$, the density being $\rho = kxy$, where k is a constant. [5+5]

OR

7.a) Evaluate $\iint \frac{dxdy}{(1+x^2+y^2)^2}$ over one of the loops of the lemniscate $(x^2+y^2)^2 = x^2 - y^2$ by changing to polar coordinates.

b) Evaluate $\int_0^1 \int_{\sqrt{x}}^1 e^{\frac{x}{y}} dydx$. [5+5]

8. Prove that:
a) $div(F \times G) = G.(curl\ F) - F.(curl\ G)$ and,
b) $curl\ (F \times G) = F(\text{div } G) - G(\text{div } F) + (G.\nabla)F - (F.\nabla)G$. [5+5]

OR

9.a) Prove the vector identity $\nabla \times (\nabla \times V) = \nabla(\nabla . V) - \nabla^2 V$.

b) Find whether $V = xyz(yz\bar{i} + xz\bar{j} + xy\bar{k})$ is irrotational, if so find the scalar potential for the vector field V. [5+5]

10.a) Evaluate by Stoke's theorem $\int_C (yzdz + zxdy + zxdz)$ where C is the curve $x^2 + y^2 = 1, z = y^2$.

b) Find the work done by the variable force $F = 2y\bar{i} + xy\bar{j}$ on a particle when it is displaced from the origin to the point $R = 4\bar{i} + 2\bar{j}$ along the parabola $y^2 = x$. [5+5]

OR

11.a) Verify Green's theorem in plane for $f = (x^2+y^2)\,\bar{i} - 2xy\,\bar{j}$ over the rectangle $x = a, x = -a, y = 0, y = b$.

b) Verify Gauss divergence theorem for $f = y\,\bar{i} + x\,\bar{j} + z^2\bar{k}$ for the surface bounded by $x^2+y^2 = 9$, $z = 0$, $z = 2$. [5+5]

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