

R16

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, September - 2023

MATHEMATICS - II

(Common to CE, ME, MCT, MMT, AE, MIE, PTM, MSNT)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

- 1.a) Find the Laplace transform of $e^{at} \sin bt$. [2]
- b) Find the Laplace transform of $v(t) = \begin{cases} t, & 0 < t < 1 \\ 1, & t > 1 \end{cases}$. [3]
- c) Find the value of $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta$? [2]
- d) Find the value of $\int_0^1 x^3 (1 - \sqrt{x})^5$ using Beta and Gamma functions. [3]
- e) Find the area bounded by the curves $y = 0, y = 1, x = 1$ and $x + y = 2$? [2]
- f) Find the double integral of $f(x, y) = \frac{x}{y}$ over the region in the first quadrant bounded by the lines $y = x, y = 3x, x = 2$ and $x = 3$? [3]
- g) Evaluate $\text{div } \vec{F}$ at the point $(1, 2, 3)$ given $\vec{F} = x^2 yz \vec{i} + xy^2 z \vec{j} + xyz^2 \vec{k}$. [2]
- h) Evaluate $\text{curl } \vec{F}$ at the point $(1, 2, 3)$ given $\vec{F} = 3x^2 \vec{i} + 5xy^2 \vec{j} + 5xyz^3 \vec{k}$. [3]
- i) Show that the vector $(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational. [2]
- j) Find the area bounded by the curves, $y = 1, x = 1, x = e^y$ and $x = e$. [3]

PART - B

(50 Marks)

- 2.a) Prove that $L^{-1} \left\{ \frac{2a^2 s}{s^4 - a^4} \right\}$ by using the convolution theorem. [5+5]
- b) Solve an ordinary differential equation $y'' + y' = \cos t$ when $y(0) = 0 = y'(0) = 0$. [5+5]

OR

- 3.a) Find the Laplace transform of $f(t) = t/T$ for $0 \leq t \leq T$ and $f(t)$ is periodic with period T .

- b) Find $L^{-1} \left\{ \frac{a^3}{s^2 (s^2 + a^2)} \right\}, s > 0$. [5+5]

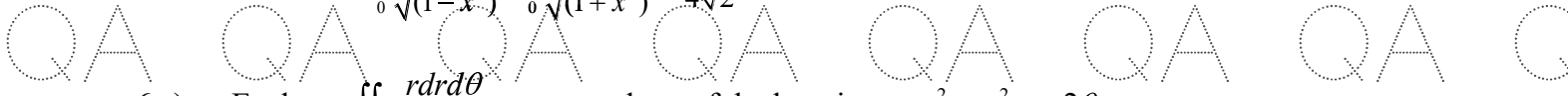
- 4.a) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

- b) Evaluate $\int_0^{\pi/2} \sqrt{\tan x} dx$. [5+5]



OR

5. Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$. [10]



6.a) Evaluate $\iint \frac{rdrd\theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2 = a^2 \cos 2\theta$.

b) Using double integration, find the Centre of Gravity of a lamina in the shape of a quadrant of the curve $(x/a)^{2/3} + (y/b)^{2/3} = 1$, the density being $\rho = kxy$, where k is a constant.

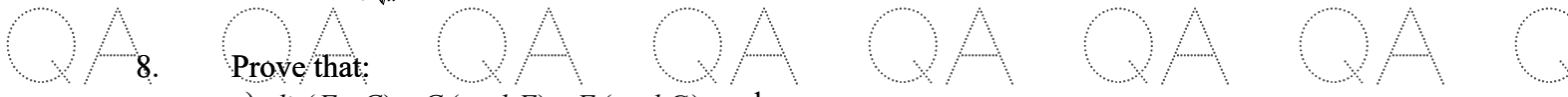
[5+5]



OR

7.a) Evaluate $\iint \frac{dx dy}{(1+x^2+y^2)^2}$ over one of the loops of the lemniscate $(x^2+y^2)^2 = x^2 - y^2$ by changing to polar coordinates.

b) Evaluate $\int_0^1 \int_{\sqrt{x}}^1 e^y dy dx$. [5+5]



8. Prove that:

a) $\text{div}(F \times G) = G \cdot (\text{curl } F) - F \cdot (\text{curl } G)$ and,

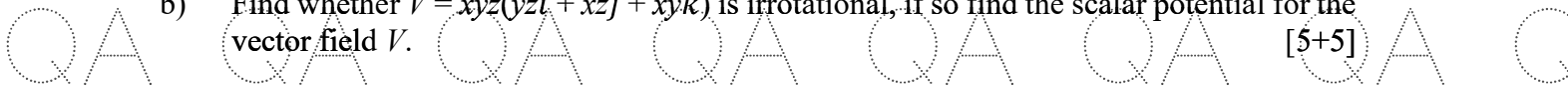
b) $\text{curl}(F \times G) = F(\text{div } G) - G(\text{div } F) + (G \cdot \nabla)F - (F \cdot \nabla)G$.

[5+5]

OR

9.a) Prove the vector identity $\nabla \times (\nabla \times V) = \nabla(\nabla \cdot V) - \nabla^2 V$.

b) Find whether $V = xyz(y\bar{i} + xz\bar{j} + xy\bar{k})$ is irrotational, if so find the scalar potential for the vector field V . [5+5]



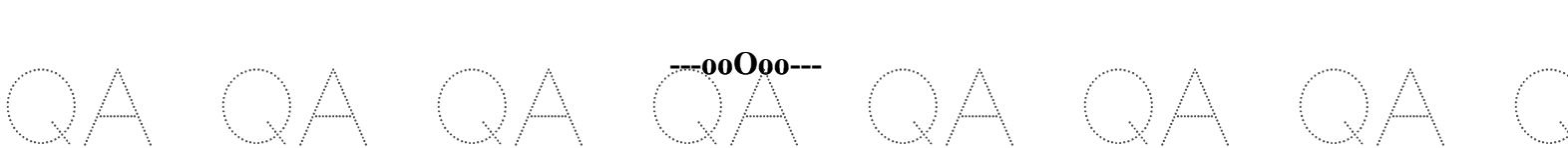
10.a) Evaluate by Stoke's theorem $\int_C (yzdz + zxdy + xzdx)$ where C is the curve $x^2 + y^2 = 1, z = y^2$.

b) Find the work done by the variable force $F = 2y\bar{i} + xy\bar{j}$ on a particle when it is displaced from the origin to the point $R = 4\bar{i} + 2\bar{j}$ along the parabola $y^2 = x$. [5+5]

OR

11.a) Verify Green's theorem in plane for $f = (x^2+y^2)\bar{i} - 2xy\bar{j}$ over the rectangle $x = a, x = -a, y = 0, y = b$.

b) Verify Gauss divergence theorem for $f = y\bar{i} + x\bar{j} + z^2\bar{k}$ for the surface bounded by $x^2+y^2 = 9, z = 0, z = 2$. [5+5]



---ooOoo---

